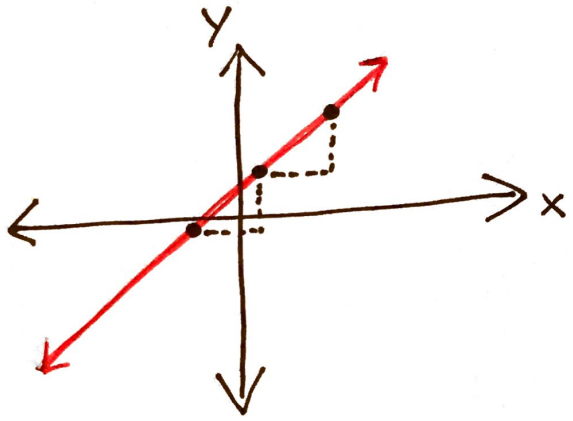
The Derivative

- Rate of Change
  - slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{Rise}}{\text{Run}}$

Ex.



constant slope

$y = mx + b$

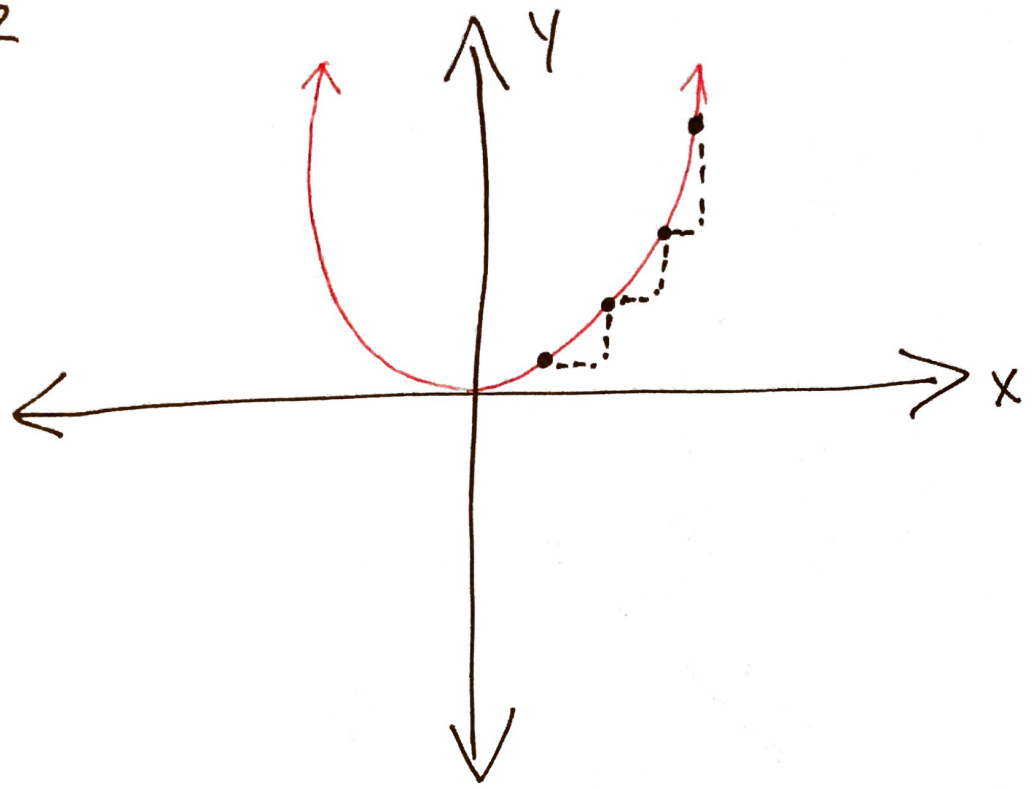
Slope $= m$

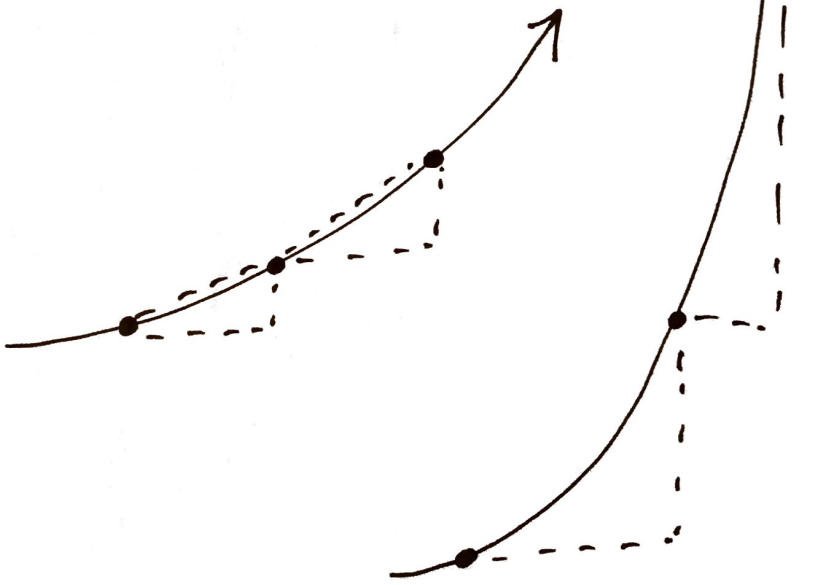
[Linear Function: slope is always a constant]

Ex. $y = 2x + 3$ $\qquad$ $y = -\frac{1}{2}x + 17$

$m = 2$ $\qquad$ $m = -\frac{1}{2}$

$y = x^2$





Ex. $y = 2x + 3$

$y' = 2$

Ex. $2x^3 + 4x + 3$

$6x^2 + 4$

Ex. $7x^4 + 3x^2 - x + 10$

$28x^3 + 6x - 1$

Properties of Slope

- $m = \frac{y_2 - y_1}{x_2 - x_1}$ = slope

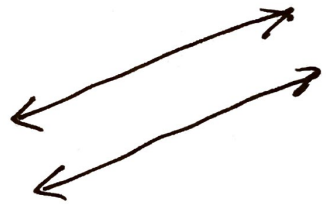
- $y - y_1 = m(x - x_1)$ = point slope

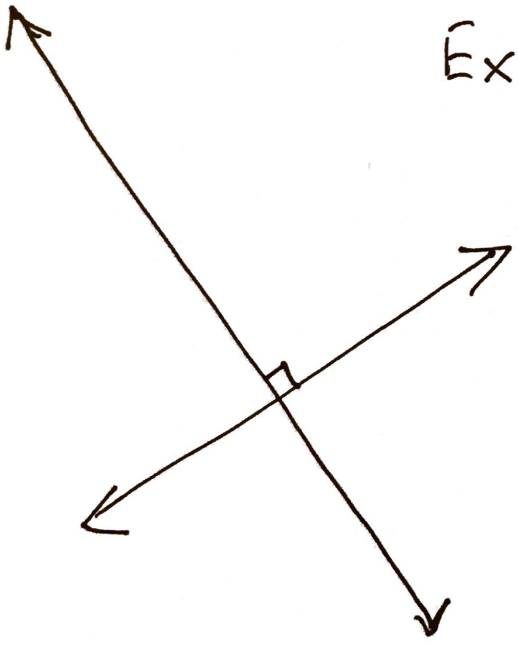
- Parallel: $m_1 = m_2$

- Perpendicular: $m_1 = -\frac{1}{m_2}$ (Perp)

Ex. $y = 2x + 3$

$y = 2x + 9$



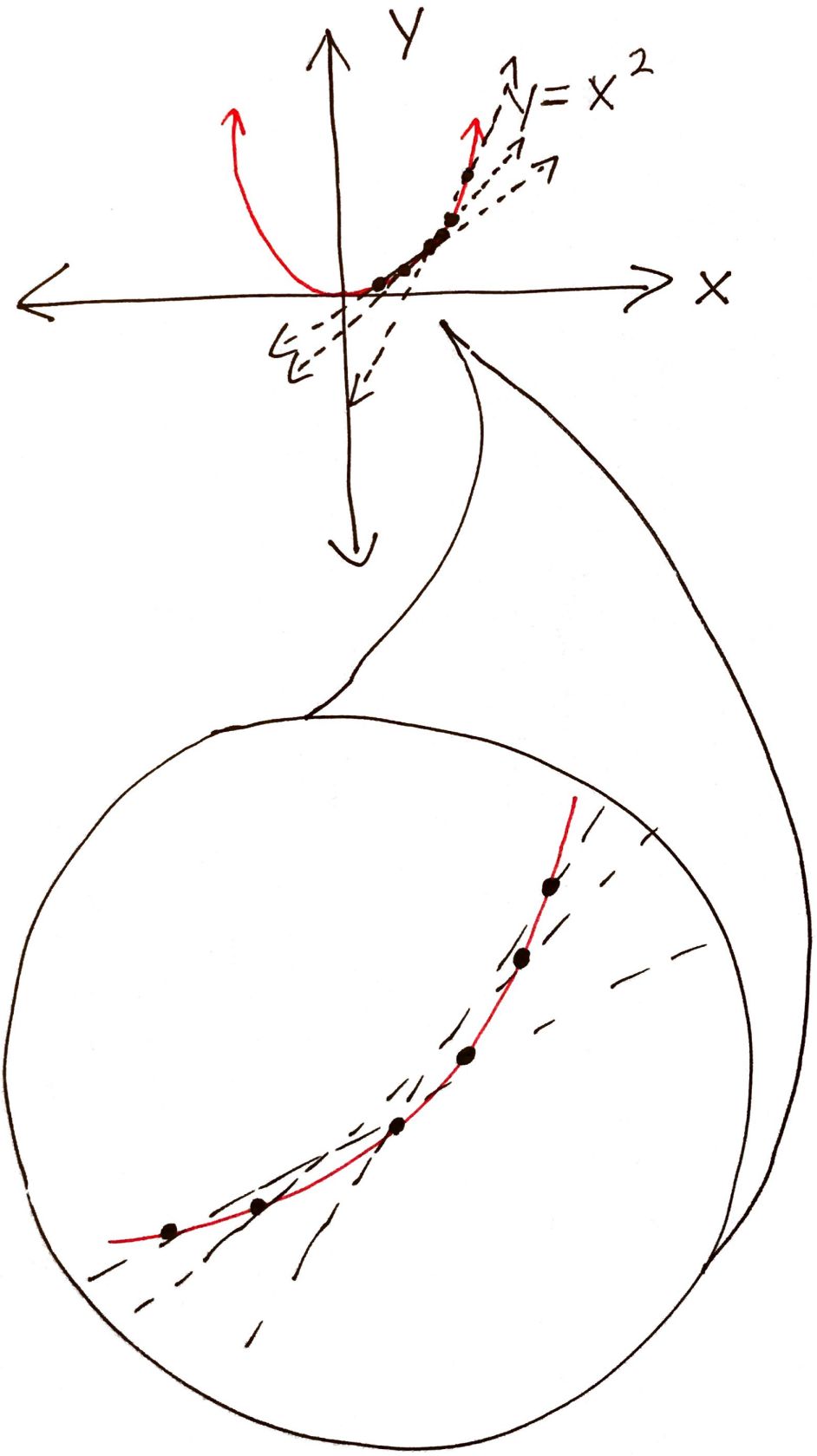


Ex. $y = 3x + 1$

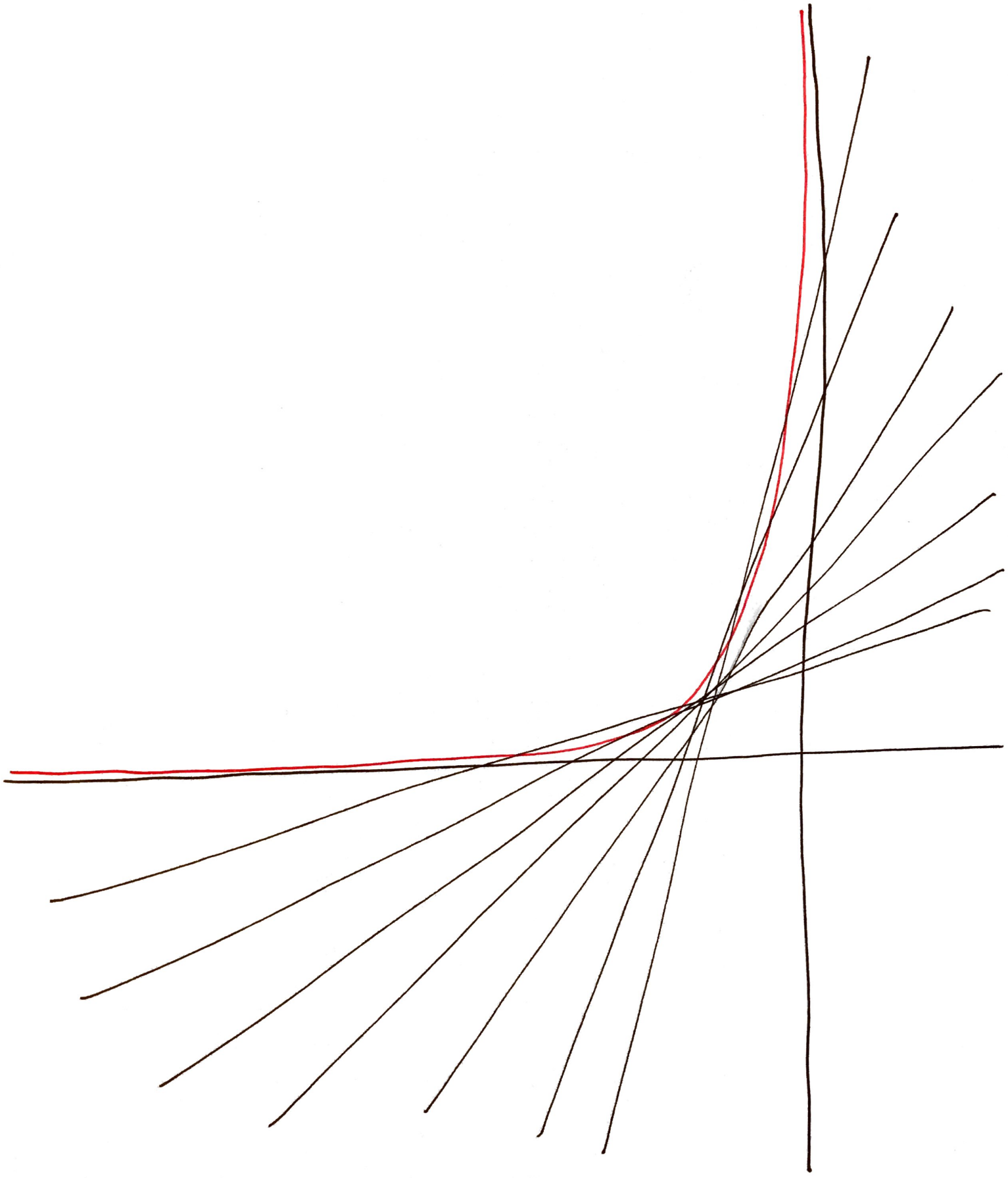
$y = -\frac{1}{3}x + 5000$

Point-Slope Form

"Tangent Line" = $y - y_1 = m(x - x_1)$



Diagram



Defining Derivative

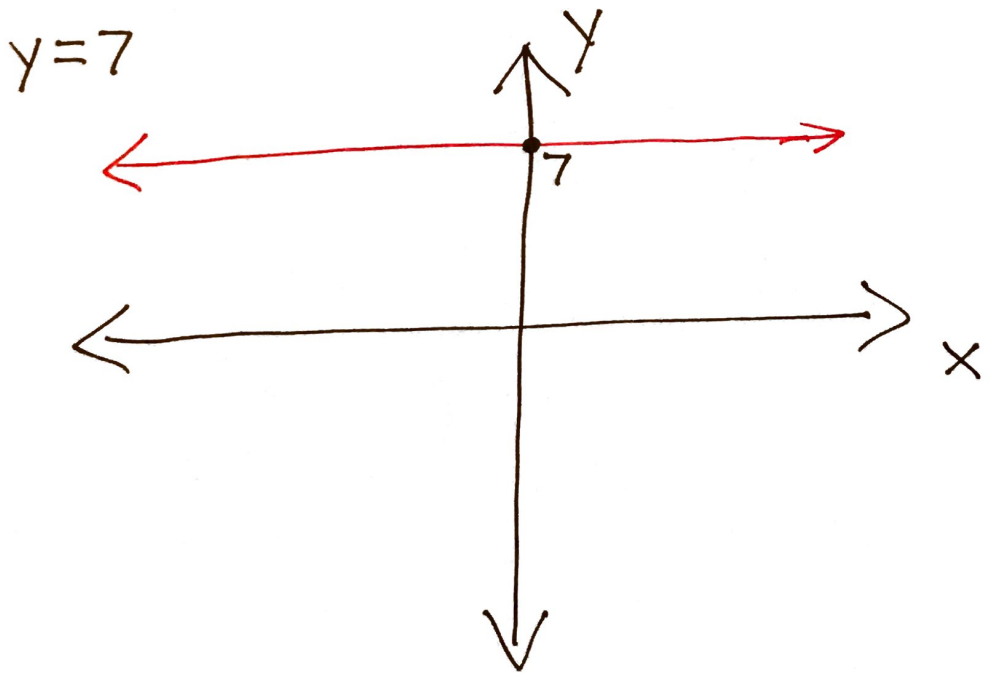
- Computing $f'(x)$ or f "prime" of x
  - denoted $f'(x)$, f', or $\frac{d}{dx}$

Def If $f(x) = mx + b$, then:

$$f'(x) = m \quad \text{[our slope!]}$$

- Constant Function: $f(x) = b$

$$f'(x) = 0$$

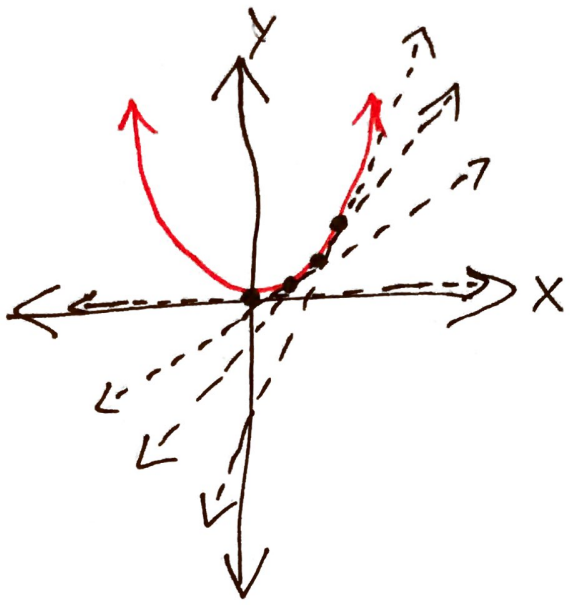


$\frac{\text{rise}}{\text{run}} : \frac{0}{\#}$

0

x	y	
-2	4	> 3
-1	1	> 1
0	0	> 1
1	1	> 3
2	4	
3	9	> 5
		> 7
4	16	
⋮		

$f(x) = x^2$



$f(x) = x^2$

$f'(x) = 2x$, derivative @ a point:

$x = 0$ slope = 0
$x = 1$ " " = 2
$x = 2$ " " = 4
$x = 3$ " " = 6
⋮

Power Rule: $f(x) = x^r$

$f'(x) = r x^{r-1}$

↳ If # out front, then $r \cdot \#$

Ex. $f(x) = x^2$

$f'(x) = 2x^1 = 2x$

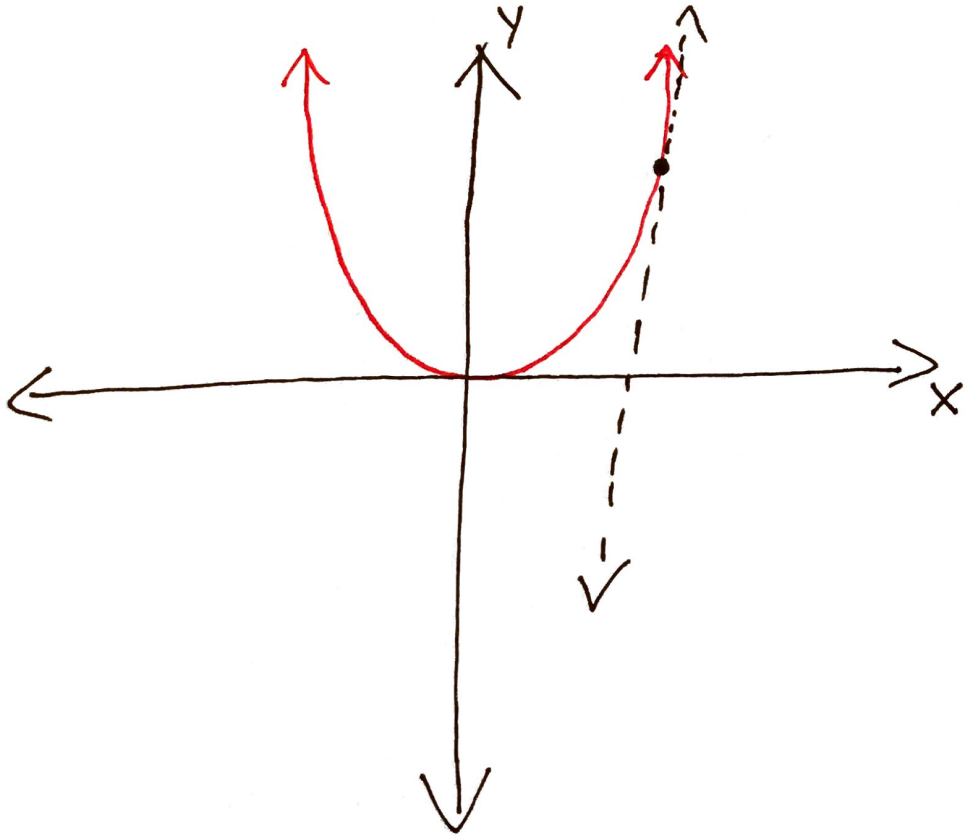
Derivative (Cont'd)

- $f'(x) = m$
- $f'(x) = r \cdot \# x^{r-1}$

Ex. $3x^4$

$m = f'(x) = 3 \cdot 4 x^3$

$= 12 x^3$



x	y
0	0
1	12
2	96
3	324

↓

Real fast!

Practice

1. $2x^3 + 3$

$2 \cdot 3x^2 + 0$

$6x^2$

2. $4x^5$

$4 \cdot 5x^4$

$20x^4$

3. $x^3 + x + 1$

$3 \cdot 1x^2 + 1x^0 + 0$

$3x^2 + 1$

4. $2x^7$

$2 \cdot 7x^6$

$14x^6$

5. $x^{-3} + 1$

$-3 \cdot 1\, x^{-4}$

$-3x^{-4}$

or

$$\frac{-3}{x^4}$$

6. $-2x^4$

$-2 \cdot 4\, x^3$

$-8x^3$