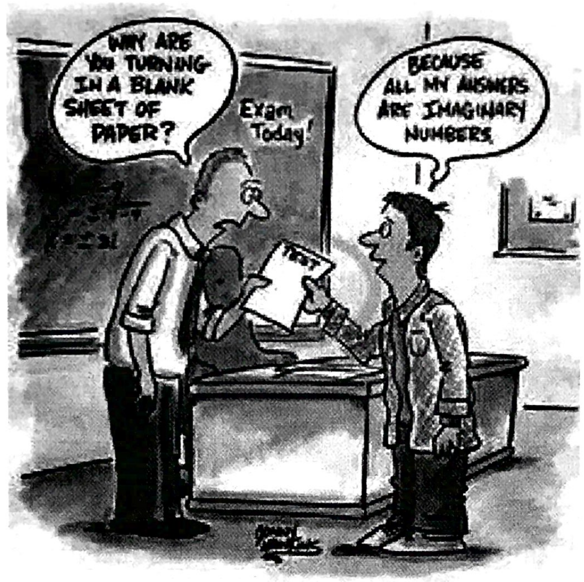
Complex Numbers – Imaginary Numbers with Reals

NAME: ______________________________ DATE: _______________

WEEK: _______________ PTS. EARNED: _________



The imaginary number is based off the concepts of the square root or $\sqrt{b}$ where b is the base.

Square root is determined by: __________ x ___________ = b

For example: $\sqrt{9}$ = what times itself gives us 9? So $\underline{3}$ x $\underline{3}$ = 9, therefore $\sqrt{9} = 3$.

Do Now: $\sqrt{81}$ $\sqrt{16}$ $\sqrt{27}$

*** Sometimes we may have to break the "radical" into its components or square roots that we already know to simplify. ***

Now consider the following: $\sqrt{-4}$

____________ x ____________ = -4

Question to ask: What times itself gives us a negative 4? In fact, what times itself gives us a negative number? – Consider the example of squaring a number, $x^2$and whether or not we get a negative number.

So even though we call "imaginary numbers" imaginary, there were always there, just not yet discovered – since we already had the term "numbers" mathematicians considered using imaginary since we did not see them clearly from the start.

To make sense of square root of a negative number, we created a system (imaginary) or a language to express these numbers.

The ***imaginary number***, denoted i is equal to the negative square root of 1:

$$i = \sqrt{-1}$$

So for example: $\sqrt{-4}$

In reality, we are replacing the negative sign with a letter i denoting the imaginary number and then determining what the square root is of the real number itself.

Exercise Problems

$\sqrt{-25}$ $\sqrt{-36}$ $\sqrt{-27}$

$\sqrt{-7}$ $\sqrt{-9}$ $\sqrt{-81}$

A ***Complex Number*** includes a part that is real and a part that is imaginary; hence – complex!

The form of a complex number is $a + bi$ where a and b are real numbers and i is the imaginary unit itself.

Adding / Subtracting: To add or subtract complex numbers, simply combine the reals together and the imaginary together:

$(5 + 8i) + (3 - 2i)$ $(2 - 7i) + (5 + 2i)$ $(3 + \sqrt{-4}) + (11 - \sqrt{-9})$

To multiply two complex numbers, just use FOIL (or distributive property):

NOTE: $i^2 = \left(\sqrt{-1}\right)^2 = -1$

$(5+8i)(3-2i)$

$(6-i)(4+5i)$

$(7+2i)(7-2i)$

$(1+8i)^2$

These are very helpful when working with square roots that have negative numbers, just like the possibility of a negative number using the quadratic formula:

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Consider: $\sqrt{b^2-4ac}$:

So let's look for a pattern – if $i = \sqrt{-1}$, then we have the following:

i	Square Root Notation	Exponent Notation	Equal to (=)
i^0	$\left(\sqrt{-1}\right)^0$	$\left((-1)^{\frac{1}{2}}\right)^0$	1
i^1			
i^2			
i^3			
i^4			
i^5			

We can use this pattern when determining higher end problems like the following:

i^2 i^4 $2i^2$ $10i^4$

i^{10} Challenge: i^{30}